Early Trigonometry

Calculations

used to

Determine Distances

Across Impassable Obstacles

during the early

Government Land Surveys

1858

Jerold F. Penry, LS

During the early government land surveys in Nebraska, the deputy surveyors occasionally had to determine the distance across bodies of water or areas that could not be measured by staying on a straight line. In the case of crossing rivers or lakes, when an offset line was not an option, trigonometry had to be used. It was also not always feasible to establish a right angle triangle to easily determine the required distance. Trigonometry involving oblique triangles was then used, but this required a higher degree of math. Usually, only the deputy surveyor was capable of doing such calculations. Without the aid of modern calculators to assist in the calculations, the surveyor carried a book of logarithms. Using logarithms required only having to perform addition and subtraction.

The example that follows are notes taken by U. S. Deputy Surveyor Charles A. Manners while surveying the Base Line between the states of Kansas and Nebraska in 1858. In a stretch of three miles, the crew had to cross the Republican River four times. Normally this river would not have required the use of trigonometry, but during the time of the survey it was at flood stage and was 5′ to 8′ deep.

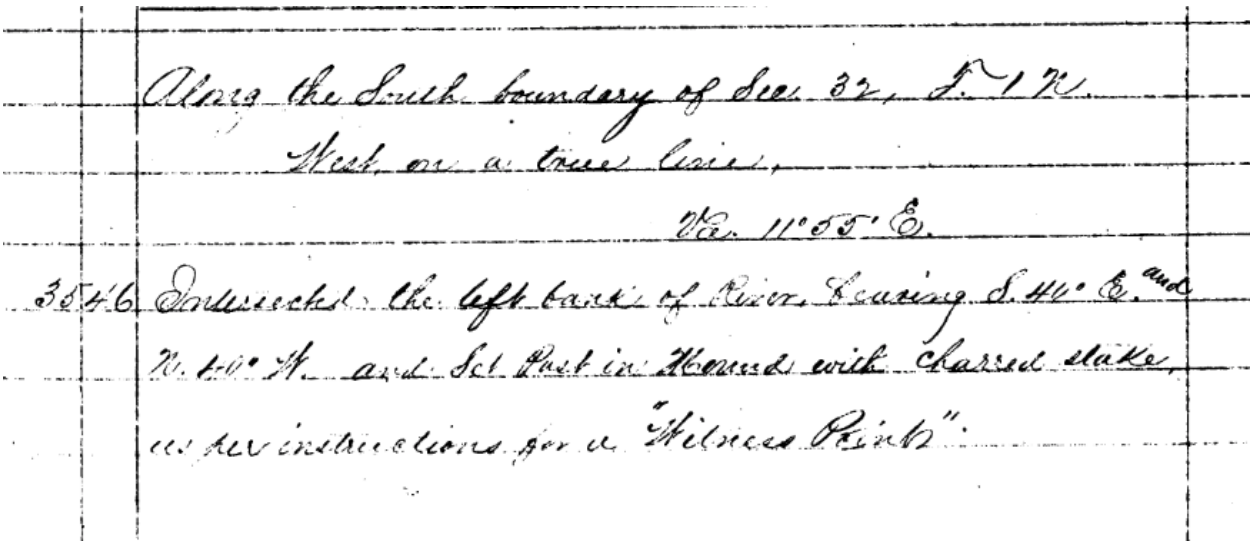
The usual method in using trigonometry was to first establish a witness point on the near side of the non-navigable river where the instrument was then set up over this point. (A "witness point" is the correct term used instead of "meander corner" when the river was not navigable or meandered). The flagman was sent across the river where he established another witness point on the opposite bank. This second point was placed on the surveyed line. From the instrument point, a baseline was established either upstream or downstream along the bank where a temporary point was placed at the far end of the baseline. The baseline was accurately measured with the chain.

While sighting the flagman on the opposite side of the river, the instrumentman turned the angle to the temporary point at the opposite end of the baseline. The instrumentman then moved to the temporary point at the far end of the baseline, sighted the flagman across the river, and turned the angle to the point on the near side of the river where he was initially set up. The crew now had two measured angles and one measured distance of the oblique triangle which was sufficient to determine the other unknown distances and angles through trigonometry using logarithm tables.

The following notes are the Original Government Field Notes of Charles A. Manners describing the four crossings of the Republican River.

(The retyped notes follow these pages).

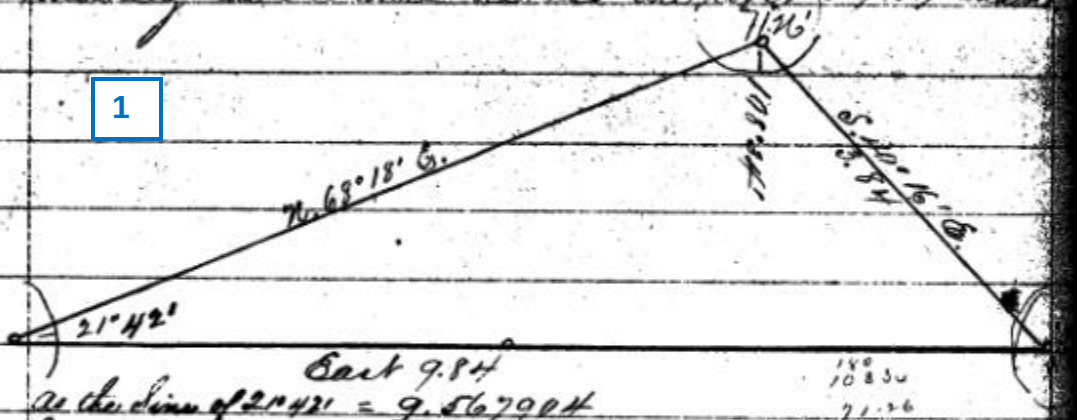
Those notes in blue are for the example at the end.



Along the South boundary of Sec 32, T. 1 N.
West on a true line,
Va. 11°55' E.
3546 Intersected the left bank of River, bearing S. 46° E. and
N. 46° W. and Set Post in Mound with charred stake,
as per instructions for a "Witness Point".

Base Line, Range 6 West

To obtain the distance across the river, I caused a flag to be set on line on the right bank, and then from the Witness Point on the left bank, measured a base N. 40° 16' W. 3.84 Chains to a Station, from which the flag on the right bank bears S. 68° 18' W. making the distance across the river = 9.84 Chains



As the Sine of 21° 42' = 9.567904
So to the Sine of 108° 34' = 9.976787
So is the Base 3.84 = 0.584331
10.561118
9.567904

180°
108 34
71.26

To dist across = 9.84 Chains = Log. 0.993214.

35.46 added to 9.84, makes 45.30 Chains

40 30 Intersected the right bank of the river, bearing N. 35° W. and S. 35° E. 5.30 Chains West of the ¼ sec. Cor., at which point I set Post for a Witness Point, and also, for a Witness to the ¼ sec. Cor. from which Post A Willow, 5 in. dia. bears S. 27° E. 29 lks. dist.

Base Line. Range 6 West.

		A 6 ~~Cottonwood~~ 20 in. dia. bears N. 35° W. 249 Lks. dist.
80	00	Set post in Mound and drove charred stake, as per instructions for cor. to secs. 31 & 32,
		Land level bottom Prairie - Soil sandy 2nd rate
		Scattering Cottonwood & Willow trees skirting the banks of the Stream.
		Along the South boundary of Sec. 31, T. 1 N.
		West, on a true line,
		Va. 11° 00' E.
40	00	Set post in Mound and drove charred stake, as per instructions for 1/4 sec. cor.
46	34	Intersected the right bank of River, bearing N. 10° E. and S. 10° W. and Set a post in Mound and drove charred stake as per instructions for a "Witness point"
		To obtain the distance acrofs the river, I set a flag in line on the left bank, and from Witness Point on the right, measured a line South 5.00 chains to a Station, from which the flag on the left bank bears N. 53° 18' W. = making the distance across the river 6.71 chains

2

Base Line. Range 6 West

Nat. Tang: 53° 18' = 1.3416009
5

Distance across River = 6.7080 1.45.

46.34 added to 6.71 makes 53.05 chains

53.05 Intersected the left bank of River, bearing N. 10° E. and S. 10° W. and Set a Post for a Witness point, from which A Cottonwood, 9 in. dia. bears N. 89° W. 31 lks. dist.
A Willow, 3 in. dia. bears S. 2° W. 17 lks. dist.
Enter Timber on left bank of River, bearing with bank.

60.00 Leave timber and enter Prairie, bearing N. 30° E. & S. 30° W.

80.00 Set post in Mound and drove charred stake as per instructions for cor. to Township one N. Ranges 6 & 7 W.
Land level bottom - Soil sandy 2d rate.
Timber, Cottonwood and Willow.

Base Line. Range 7 West

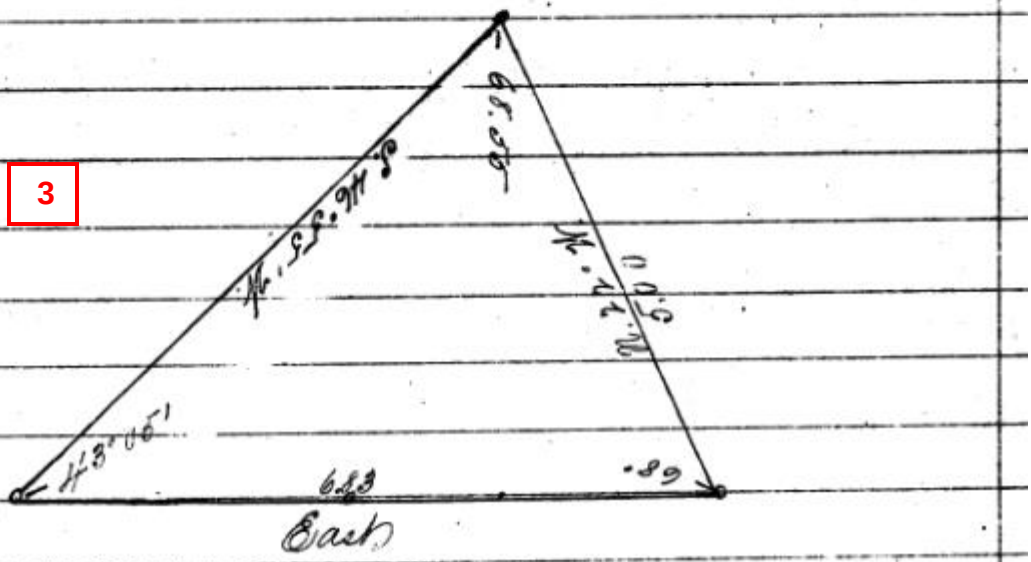
Along the South boundary of sec. 36. T. 1 N.

West on a true line.

Va. 11°50' E.

26 30 Intersected left bank of river, bearing N. 25° W. and S. 25° E. and Set Post in Mound and drove Charred stake as per instructions for a Witness Point

To obtain distance across river, I set a flag on line on the right bank, and then from the Witness Point on left bank of River, measured a Line N. 22° W. 500 lks. to a station, from which the flag on right bank bears S. 46° 55' W. - Making distance across on line, 6.83 chains



Base Lines, Range 7 West.

As Lines 48° 15' = 9.8344 60

Do to Lines 68° 55' 9.9699 09

Sine Diff Ch. Log = 0.6989 70
10.6684 79
9.8344 60

To dist across 683 Log = 0.8344 19

683 added to 26.30 makes 33.13 Chains

33 13 Intersected right bank of river, bearing N. 25° W. and S. 25° E. and Set post for a Witness Point, from which A Willow, 5 in. dia. bears N. 60° W. 7 Lks. dist.
A Willow, 5 in. dia. bears S. 64° E. 18 Lks. dist.
Compared measuring Chains with Standard and found them Correct. July 18, 1858.

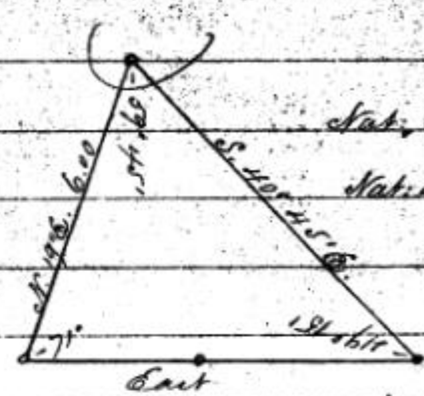
1 00 Set post in Mound and drove Barred stake, as per instructions for 1/4 sec. cor.

60 00 Leave Prairie and enter Timber, bearing S. W. & N. 60° E.

74 00 Intersected right bank of River again, bearing N. 25° E. and S. 25° W. and Set post with Barred stake, as per instructions, for a Witness point, from which
A Cottonwood, 14 in. dia. bears N. 20° E. 28 Lks. dist.
A Cottonwood, 10 in. dia. bears S. 20° W. 22 Lks. dist.
To obtain distance across river, I produced

Base Line, Range 7 West.

the line to the left bank, and at this point of Intersection, set a temporary post, from which I measured a base N. 19° E. 600 chains to a station, from which the Post at the Witness Point on right bank, bears S. 40° 45' E. — making the distance across river = 6.84 chains.



Nat. Sine 49°15' = .7575650

Nat. Sine 59°45' = .8638355

.7575650 : .8638355 :: 600 lks.

```
                 600
.7575650) 518.3013000 (684 links = distance across
          45453900
           63762300
           60605200
            31571000
            30302600
             1268400
```

6.84 chains added to 77.00 chains, makes 83.84 chains.
Consequently the post on left bank of river is 3.84 chs. West of Cor. to secs. 35 & 36, Hence at

80 00 Corner in river — inaccessible.

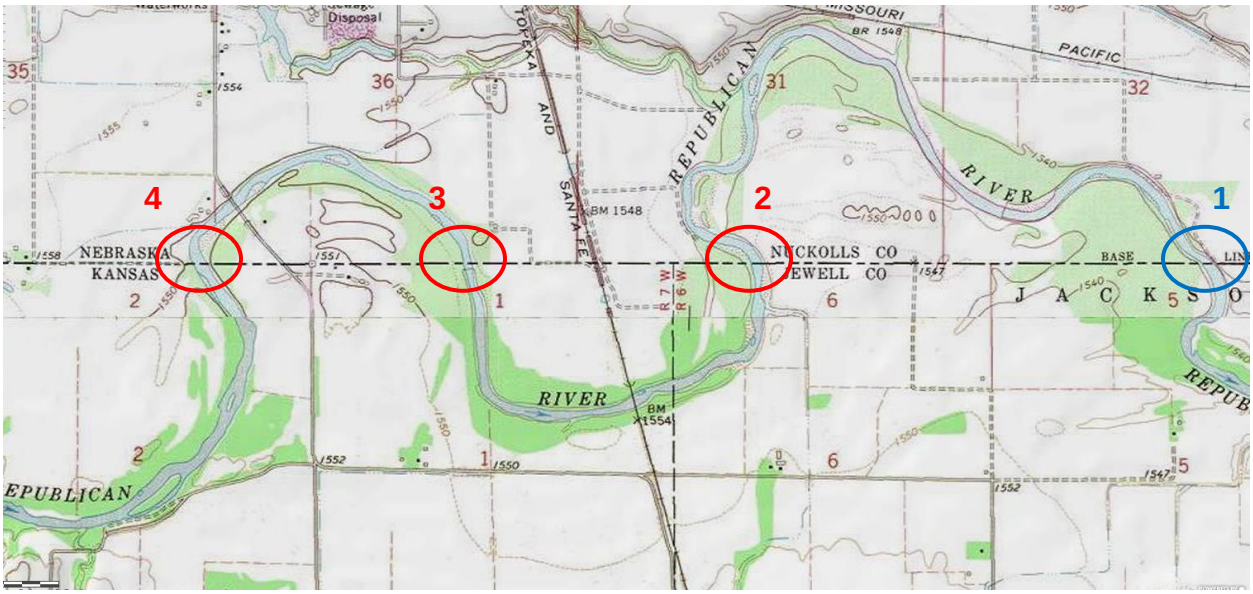
Base Line, Range 7 West

As the Corner Cannot be located in its true place
I select the point where the temporary posts stands
on the left bank of River, as the nearest suitable
Witness point, and there Set post with charred stake
as per instructions, 384 lks. West of the true corner
for a Witness Corner to secs. 35 & 36, from which
A Cottonwood, 30 in. dia. bears N. 20° E. 213 lks. dist.
A Cottonwood, 8 in. dia. bears S. 7½° W. 117 lks. dist.
A Cottonwood, 10 in. dia. bears S. 16° E. 42 lks. dist.
A Cottonwood, 8 in. dia. bears N. 45° E. 61 lks. dist.
Land level sandy bottom - Soil 2nd rate.
Timber, chiefly Cottonwood, open woods.

Along the South boundary of sec. 35. T. 1 N.
West, on true line,

Va. 11°50' E.

3 84 Intersected the left bank of river, bearing N. 20° E.
and S. 20° W. at the Witness cor. to secs. 35 & 36,
Leave Timber & enter prairie, bearing with the bank
40 00 Set post in Mound and drove charred stake, as per
instructions for 1/4 sec. cor.



Topographic map of the Republican River at Superior, Nebraska.

Along the South boundary of Sec. 32, T1N, [R6W].

West on a true line.

Va. 11°55′ E.

35.46 [chains] – Intersected the left bank of River, bearing S. 40° E. and N. 40° W. and Set Post in Mound with charred stake as per instructions for a "Witness Point".

Base Line. Range 6 West

To obtain the distance across the river, I caused a flag to be set in line on the right bank and then from the Witness Point on the left bank measured a base N. 40°16′ W., 3.84 chains to a Station from which the flag on the right bank bears S. 68°18′ W., making the distance across the river = 9.84 chains.

[Diagram]

As the Sine of 21°42′ =	*9.567904*
Is to the Sine of 108°34′ =	*9.976787*
So is the Base 3.84	*0.584331*
	10.561118
	9.567904

To dist across = 9.84 chains = Log. 0.993214.

35.46 added to 9.84 makes 45.30 chains.

45.30 [chains] – Intersected the right bank of the river bearing N. 35° W. and S. 35° E.,
5.30 chains West of the ¼ sec. cor. at which point I set Post
for a Witness Point and also for a Witness to the ¼ sec. cor. from which post
A Willow 5 in. dia. bears S. 27° E, 29 links distant.

Base Line. Range 6 West

A Cottonwood 20 in. dia. bears N. 35° W., 249 links distant.

80.00 [chains] – Set post in mound and drove charred stake, as per instructions
for cor. to sections 31 & 32.
Land level bottom prairie. Soil sandy 2nd rate.
Scattering Cottonwood & Willow trees skirting the banks of the Stream.

Along the South boundary of Sec. 31, T1N, [R6W].
West on a true line.
Va. 11°50′ East
40.00 [chains] – Set post in mound and drove charred stake, as per instructions for ¼ sec. cor.
46.34 [chains] – Intersected the right bank of river bearing N. 10° E., and S. 10° W. and Set a post in Mound and drove charred stake as per instructions for a "Witness point".
To obtain the distance across the river, I set a flag in line on the left bank and from Witness Point on the right measured a base South 5.00 chains to a Station from which the flag on the left bank bears N. 53°18′ W., making the distance across the river 6.71 chains.

Base Line. Range 6 West

Nat. Tang: 53°18′ = 1.341602
5
Distance across river = 6.7080145
46.34 added to 6.71 makes 53.05 chains.
53.05 [chains] – Intersected the left bank of River bearing N. 10° E. and S. 10° W. and Set a Post for Witness point from which
A cottonwood, 9 in. dia. bears N. 89° W., 31 links distant.
A willow 3 in. dia. bears S. 2° W., 17 links distant.
Enter timber on left bank of River bearing with bank.
60.00 [chains] – Leave timber and enter Prairie bearing N. 30° E. and S. 30° W.
80.00 [chains] – Set post in mound and drove charred stake as per instructions for cor. to Township one North, Ranges 6 & 7 West.
Land level bottom – Soil sandy 2nd rate.
Timber, Cottonwood and Willow.

Base Line. Range 7 West

Along the South boundary of Sec. 36, T1N [R7W].
West on a true line.
Va. 11°50′ E.

26.30 [chains] – Intersected left bank of river bearing N. 25° W. and S. 25° E. and Set Post in Mound and drove charred stake as per instructions for a Witness Point.
To attain distances across river, I set a flag in line on the right bank and then from the Witness Point on left bank of river, measured a base N. 22° W. , 5.00 chains to a station from which the flag on right bank bears S. 46° 55′ W., making distance across on line 6.83 chains.

[Diagram]

Base Line. Range 7 West

As Sine is 43°05′ =	*9.834460*
Is to Sine 68°55′	*9.969909*
So is 5.00 ch. Log: =	*0.698970*
	10.068879
	9.834460
To dist across 6.83 Log =	*0.834419*

6.83 added to 26.30 makes 33.13 chains
33.13 [chains] – Intersected right bank of river bearing N. 25° W. and S. 25° W. and Set post for a Witness Point from which
A Willow 5 in. dia. bears N. 60° W., 7 links distant.
A Willow 5 in. dia. bears S. 64° E., 18 links distant.
Compared measuring chains with Standard and found them correct. July 18, 1858.

40.00 [chains] – Set post in Mound and drove charred stake as per instructions for ¼ sec. cor.
60.00 [chains] – Leave prairie and enter Timber, bearing S.W. & N. 60° E.
77.00 [chains] – Intersected right bank of River again, bearing N. 25° E. and S. 25° W. and Set post with charred stake as per instructions for a Witness point, from which
A Cottonwood 14 in. dia. bears N. 20° E., 28 links distant.
A Cottonwood 10 in. dia. bears S. 20° W., 22 links distant.
To obtain distance across river, I produced

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the line to the left bank and at this point of Intersection, Set a temporary post from which I measured a base N. 19° E., 6.00 chains to a station, from which the Post at the Witness Point on right bank bears S. 40° 45′ E. – making the distance across river = 6.84 chains.

[Diagram]

Nat. Sine 49°15′ = .7575650
Nat. Sine 59°45′ = .8638355

.7575650 . . .8638355 : : 600 links

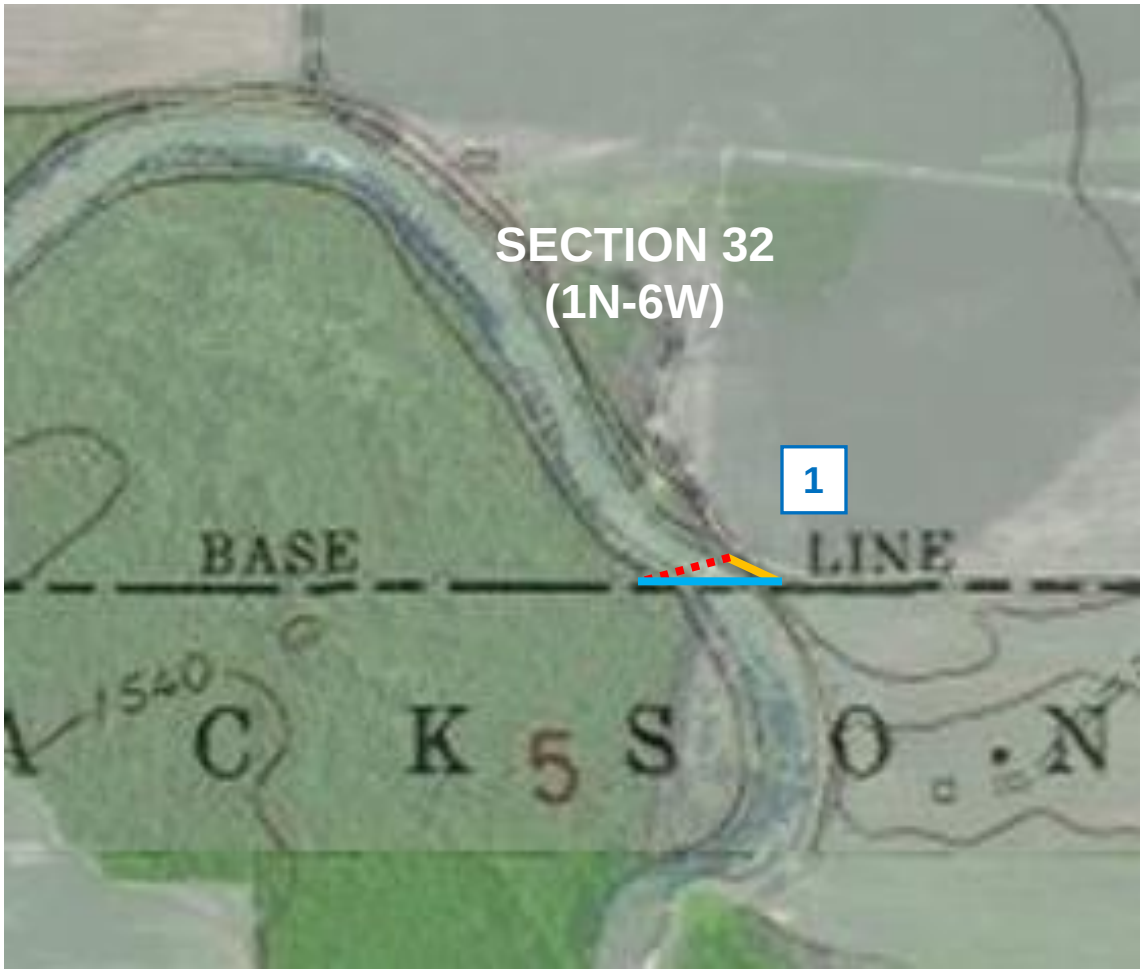
```
                    600
.7575650 ) 518.3013000  ( 684 links = distance across
           45453900
            63.762300
            60605200
             31571000
             30302600
              1268400
```

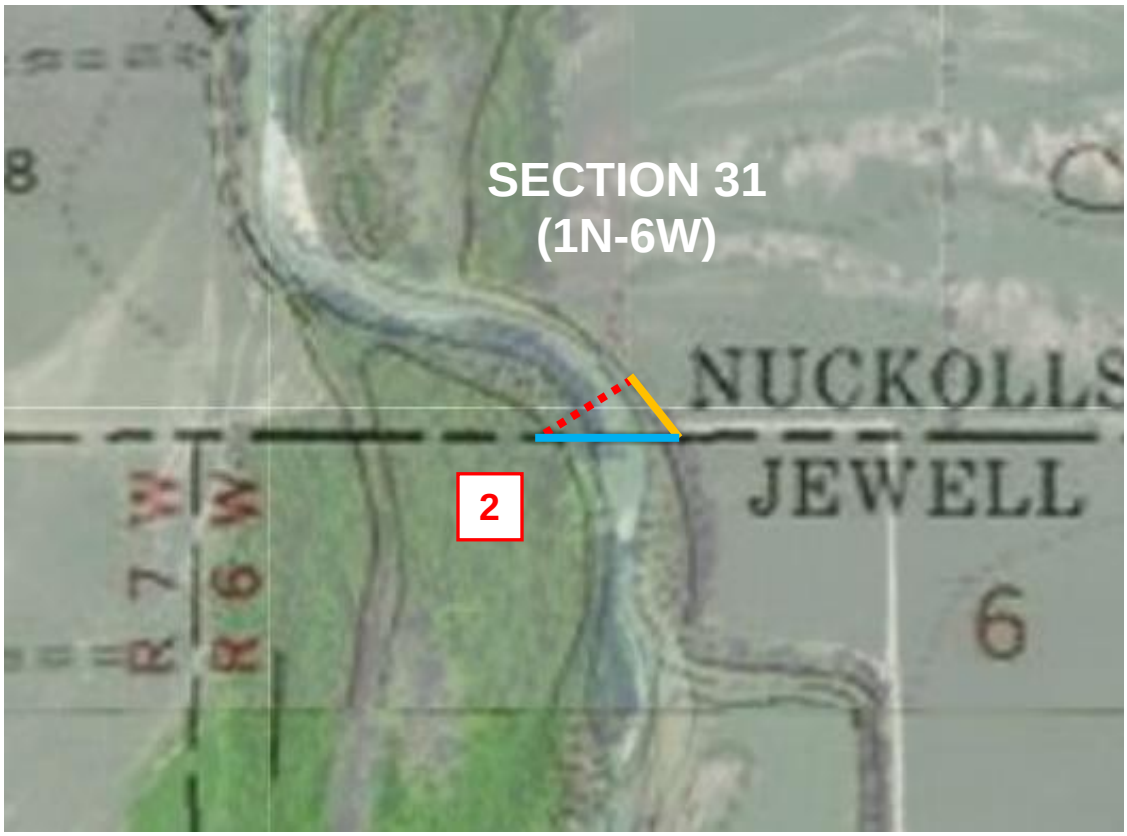
6.84 chains added to 77.00 chains makes 83.84 chains
Consequently the post on left bank of river is 3.84 chains
West of corner to sections 35 and 36, thence at
80.00 [chains] – Corner in river – inaccessible.

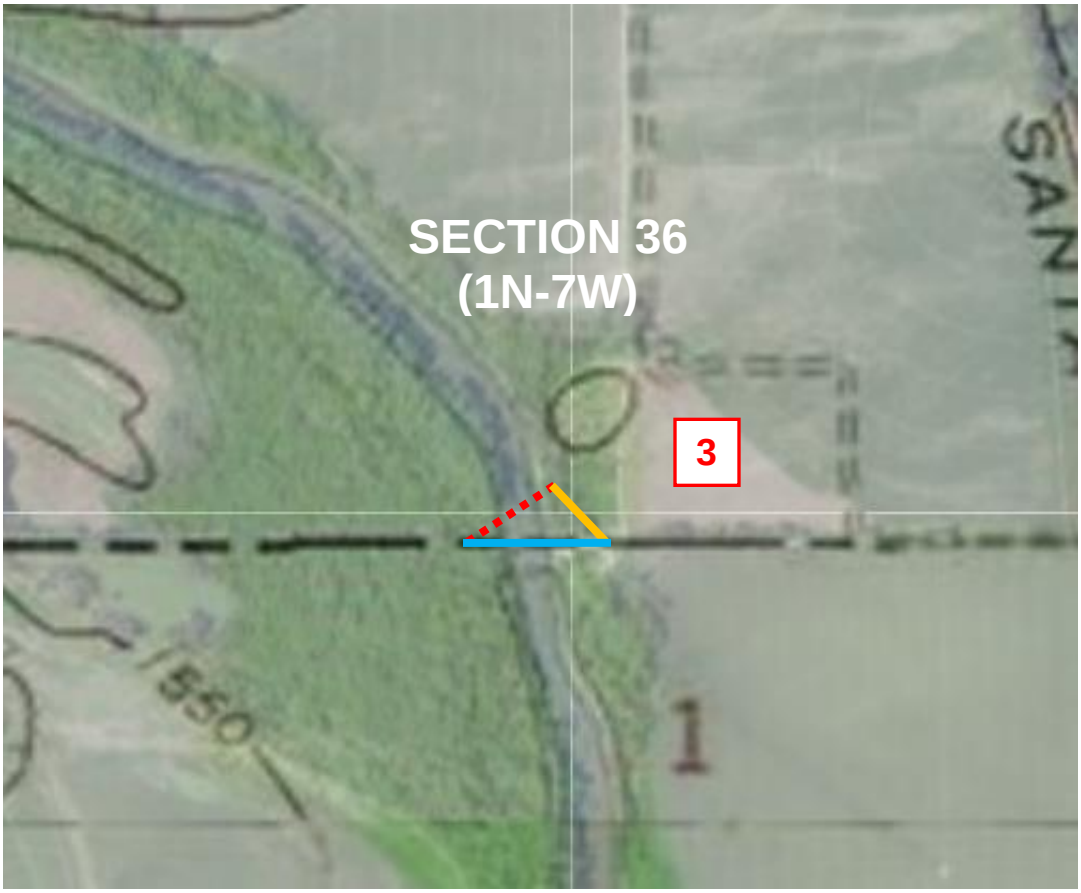
Base Line. Range 7 West

As the corner cannot be located in its true place, I select the point where the temporary post stands on the left bank of the River as the nearest suitable Witness point, and there Set post with charred stake as per instructions, 384 links West of the true corner for a Witness Corner to sections 35 & 36 from which
A Cottonwood, 20 in. dia. bears N. 20° E., 213 links distant.
A Cottonwood, 8 in. dia. bears S. 7 ½° W., 117 links distant.
A Cottonwood 10 in. dia. Bears S. 16° E., 42 links distant.
A Cottonwood, 8 in. dia. bears N. 43° E., 61 links distant.
Land level, sandy bottom – Soil 2nd rate.
Timber chiefly Cottonwood, open woods.

Along the South boundary of Sec. 35, T.1N. [R7W]
West on true line.
Va. 11°50′ E.
3.84 [chains] – Intersected the left bank of river bearing N. 20° E. and S. 20° W. at the witness corner to Sec's 35 & 36.
Leave Timber and enter prairie bearing with the bank.
40.00 [chains] – Set post in Mound and drove charred stake as per instructions for ¼ sec. cor.


SECTION 32
(1N-6W)
1
BASE
LINE
1540
C
K
5
S
O
N


SECTION 31
(1N-6W)
NUCKOLLS
JEWELL
2
6
R 7 W
R 6 W



= MEASURED BASE LINE

= RIVER CROSSING (CALCULATED)

= THIRD LEG OF TRIANGLE

River Crossing No. 1

1. Survey to the near bank of the river and set a point for a Witness Corner.

   *Chaining is now at **35.46** chains from the last section corner.*

2. Occupy the Witness Point just established.

3. Send the flagman across the river and set a point on the opposite bank of the river for a Witness Corner by sighting across the river on the surveyed line.

4. Establish and measure a baseline along the near bank of the river from the first Witness Corner.

   *Baseline measures to be **3.84** chains.*

5. While occupying the Witness Corner on the near bank which is also the first end of the baseline, sight the Witness Corner on the opposite bank and turn the angle to the temporary point at the far end of the baseline.

   *Angle = **49°44'***

6. Move the instrument to and occupy the temporary point on the far end of the baseline. Sight the other end of the baseline that is also the Witness Point on the near bank. Turn the angle between the Witness Point on the near bank to the Witness Point on the opposite bank.

   *Angle = **108°34'** (Supplementary angle = **71°26'**)*
   (Note: The supplementary angles have the same Log value).

7. Two measured angles and one measured line of a triangle are now known.

8. Compute the missing angle that is at the Witness Point at the opposite bank.

   *180° - 108°34' - 49°44' = **21°42'** (Or for a check turn the angle after crossing the river).*

9. Obtain the logarithms from tables in the book.

   *Sine 21°42' from logarithm tables = **9.567904***

   *Sine of 108°34' (or the supplementary angle 71°26') from logarithm tables = **9.976787***

   *Base distance 3.84 from logarithm tables = **0.584331***

   *9.976787 + 0.584331 = **10.561118** **(Addition)***

   *10.561118 – 9.567904 = **0.993214** **(Subtraction)***

   *Logarithm of 0.993214 = **9.84** **chains** (This is the distance across the river between the Witness Points).*

9.84 + 35.46 = **45.30** chains. *(This is the location of the Witness Corner on the opposite bank).*

TABLES

OF

LOGARITHMS OF NUMBERS

AND OF

SINES AND TANGENTS

FOR EVERY

TEN SECONDS OF THE QUADRANT,

WITH OTHER USEFUL TABLES.

BY ELIAS LOOMIS, A.M.,

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NEW YORK:

HARPER & BROTHERS, PUBLISHERS,

82 CLIFF STREET.

1848.

Min.	Sine of 20 Degrees. 0″	10″	20″	30″	40″	50″	
0	9.534052	4110	4167	4225	4283	4341	59
1	4399	4456	4514	4572	4630	4687	58
2	4745	4803	4861	4918	4976	5034	57
3	5092	5149	5207	5265	5322	5380	56
4	5438	5495	5553	5610	5668	5726	55
5	5783	5841	5898	5956	6014	6071	54
6	6129	6186	6244	6301	6359	6416	53
7	6474	6531	6589	6646	6704	6761	52
8	6818	6876	6933	6991	7048	7105	51
9	7163	7220	7278	7335	7392	7450	50
10	9.537507	7564	7622	7679	7736	7794	49
11	7851	7908	7965	8023	8080	8137	48
12	8194	8252	8309	8366	8423	8480	47
13	8538	8595	8652	8709	8766	8823	46
14	8880	8938	8995	9052	9109	9166	45
15	9223	9280	9337	9394	9451	9508	44
16	9565	9622	9679	9736	9793	9850	43
17	9907	9964	..21	..78	.135	.192	42
18	9.540249	0306	0363	0420	0477	0533	41
19	0590	0647	0704	0761	0818	0875	40
20	9.540931	0988	1045	1102	1159	1215	39
21	1272	1329	1386	1442	1499	1556	38
22	1613	1669	1726	1783	1839	1896	37
23	1953	2009	2066	2123	2179	2236	36
24	2293	2349	2406	2462	2519	2576	35
25	2632	2689	2745	2802	2858	2915	34
26	2971	3028	3084	3141	3197	3254	33
27	3310	3367	3423	3480	3536	3593	32
28	3649	3705	3762	3818	3875	3931	31
29	3987	4044	4100	4156	4213	4269	30
30	9.544325	4382	4438	4494	4550	4607	29
31	4663	4719	4776	4832	4888	4944	28
32	5000	5057	5113	5169	5225	5281	27
33	5338	5394	5450	5506	5562	5618	26
34	5674	5731	5787	5843	5899	5955	25
35	6011	6067	6123	6179	6235	6291	24
36	6347	6403	6459	6515	6571	6627	23
37	6683	6739	6795	6851	6907	6963	22
38	7019	7075	7131	7187	7242	7298	21
39	7354	7410	7466	7522	7578	7633	20
40	9.547689	7745	7801	7857	7912	7968	19
41	8024	8080	8136	8191	8247	8303	18
42	8359	8414	8470	8526	8581	8637	17
43	8693	8748	8804	8860	8915	8971	16
44	9027	9082	9138	9193	9249	9305	15
45	9360	9416	9471	9527	9582	9638	14
46	9693	9749	9805	9860	9916	9971	13
47	9.550026	0082	0137	0193	0248	0304	12
48	0359	0415	0470	0525	0581	0636	11
49	0692	0747	0802	0858	0913	0968	10
50	9.551024	1079	1134	1190	1245	1300	9
51	1356	1411	1466	1521	1577	1632	8
52	1687	1742	1798	1853	1908	1963	7
53	2018	2074	2129	2184	2239	2294	6
54	2349	2405	2460	2515	2570	2625	5
55	2680	2735	2790	2845	2900	2955	4
56	3010	3065	3121	3176	3231	3286	3
57	3341	3396	3451	3506	3560	3615	2
58	3670	3725	3780	3835	3890	3945	1
59	4000	4055	4110	4165	4219	4274	0
	60″	50″	40″	30″	20″	10″	Min.
	Co-sine of 69 Degrees.						
P. Part	1″ 6	2″ 11	3″ 17	4″ 23	5″ 28	6″ 34	7″ 39	8″ 45	9″ 51

Min.	Sine of 21 Degrees. 0″	10″	20″	30″	40″	50″	
0	9.554329	4384	4439	4494	4549	4603	59
1	4658	4713	4768	4822	4877	4932	58
2	4987	5042	5096	5151	5206	5260	57
3	5315	5370	5425	5479	5534	5589	56
4	5643	5698	5753	5807	5862	5916	55
5	5971	6026	6080	6135	6190	6244	54
6	6299	6353	6408	6462	6517	6571	53
7	6626	6680	6735	6789	6844	6898	52
8	6953	7007	7062	7116	7171	7225	51
9	7280	7334	7388	7443	7497	7552	50
10	9.557606	7660	7715	7769	7823	7878	49
11	7932	7986	8041	8095	8149	8204	48
12	8258	8312	8366	8421	8475	8529	47
13	8583	8638	8692	8746	8800	8855	46
14	8909	8963	9017	9071	9125	9180	45
15	9234	9288	9342	9396	9450	9504	44
16	9558	9613	9667	9721	9775	9829	43
17	9883	9937	9991	..45	..99	.153	42
18	9.560207	0261	0315	0369	0423	0477	41
19	0531	0585	0639	0693	0747	0801	40
20	9.560855	0908	0962	1016	1070	1124	39
21	1178	1232	1286	1339	1393	1447	38
22	1501	1555	1609	1662	1716	1770	37
23	1824	1878	1931	1985	2039	2093	36
24	2146	2200	2254	2307	2361	2415	35
25	2468	2522	2576	2629	2683	2737	34
26	2790	2844	2898	2951	3005	3058	33
27	3112	3166	3219	3273	3326	3380	32
28	3433	3487	3541	3594	3648	3701	31
29	3755	3808	3862	3915	3969	4022	30
30	9.564075	4129	4182	4236	4289	4343	29
31	4396	4449	4503	4556	4610	4663	28
32	4716	4770	4823	4876	4930	4983	27
33	5036	5090	5143	5196	5250	5303	26
34	5356	5409	5463	5516	5569	5622	25
35	5676	5729	5782	5835	5888	5942	24
36	5995						23
37	6314						22
38	6632	6685	6739	6792	6845	6898	21
39	6951	7004	7057	7110	7163	7216	20
40	9.567269	7322	7375	7428	7481	7534	19
41	7587	7640	7693	7746	7799	7851	18
42	7904	7957	8010	8063	8116	8169	17
43	8222	8275	8327	8380	8433	8486	16
44	8539	8592	8644	8697	8750	8803	15
45	8856	8908	8961	9014	9067	9119	14
46	9172	9225	9277	9330	9383	9436	13
47	9488	9541	9594	9646	9699	9752	12
48	9804	9857	9910	9962	..15	..67	11
49	9.570120	0173	0225	0278	0330	0383	10
50	9.570435	0438	0541	0593	0646	0698	9
51	0751	0803	0856	0908	0961	1013	8
52	1066	1118	1170	1223	1275	1328	7
53	1380	1433	1485	1537	1590	1642	6
54	1695	1747	1799	1852	1904	1956	5
55	2009	2061	2113	2166	2218	2270	4
56	2323	2375	2427	2479	2532	2584	3
57	2636	2688	2741	2793	2845	2897	2
58	2950	3002	3054	3106	3158	3210	1
59	3263	3315	3367	3419	3471	3523	0
	60″	50″	40″	30″	20″	10″	Min.
	Co-sine of 68 Degrees.						
P. Part	1″ 5	2″ 11	3″ 16	4″ 21	5″ 27	6″ 32	7″ 37	8″ 43	9″ 48

Logarithm for 21°42’ = 9.567904

Min.	Sine of 70 Degrees. 0″	10″	20″	30″	40″	50″		Min.	Sine of 71 Degrees. 0″	10″	20″	30″	40″	50″	
0	9·9729 86	2993	3001	3009	3016	3024	59	0	9·9756 70	5677	5685	5692	5699	5706	59
1	3032	3039	3047	3055	3062	3070	58	1	5714	5721	5728	5735	5743	5750	58
2	3078	3085	3093	3101	3108	3116	57	2	5757	5764	5771	5779	5786	5793	57
3	3124	3131	3139	3146	3154	3162	56	3	5800	5808	5815	5822	5829	5837	56
4	3169	3177	3185	3192	3200	3208	55	4	5844	5851	5858	5865	5873	5880	55
5	3215	3223	3230	3238	3246	3253	54	5	5887	5894	5901	5909	5916	5923	54
6	3261	3269	3276	3284	3291	3299	53	6	5930	5938	5945	5952	5959	5966	53
7	3307	3314	3322	3330	3337	3345	52	7	5974	5981	5988	5995	6002	6010	52
8	3352	3360	3368	3375	3383	3390	51	8	6017	6024	6031	6038	6046	6053	51
9	3398	3406	3413	3421	3428	3436	50	9	6060	6067	6074	6081	6089	6096	50
10	9·9734 44	3451	3459	3466	3474	3482	49	10	9·9761 03	6110	6117	6125	6132	6139	49
11	3489	3497	3504	3512	3519	3527	48	11	6146	6153	6160	6168	6175	6182	48
12	3535	3542	3550	3557	3565	3572	47	12	6189	6196	6203	6211	6218	6225	47
13	3580	3588	3595	3603	3610	3618	46	13	6232	6239	6246	6254	6261	6268	46
14	3625	3633	3641	3648	3656	3663	45	14	6275	6282	6289	6296	6304	6311	45
15	3671	3678	3686	3694	3701	3709	44	15	6318	6325	6332	6339	6347	6354	44
16	3716	3724	3731	3739	3746	3754	43	16	6361	6368	6375	6382	6389	6396	43
17	3761	3769	3777	3784	3792	3799	42	17	6404	6411	6418	6425	6432	6439	42
18	3807	3814	3822	3829	3837	3844	41	18	6446	6454	6461	6468	6475	6482	41
19	3852	3859	3867	3875	3882	3890	40	19	6489	6496	6503	6510	6518	6525	40
20	9·9738 97	3905	3912	3920	3927	3935	39	20	9·9765 32	6539	6546	6553	6560	6567	39
21	3942	3950	3957	3965	3972	3980	38	21	6574	6582	6589	6596	6603	6610	38
22	3987	3995	4002	4010	4017	4025	37	22	6617	6624	6631	6638	6646	6653	37
23	4032	4040	4047	4055	4062	4070	36	23	6660	6667	6674	6681	6688	6695	36
24	4077	4085	4092	4100	4107	4115	35	24	6702	6709	6716	6723	6731	6738	35
25	4122	4130	4137	4145	4152	4160	34	25	6745	6752	6759	6766	6773	6780	34
26	4167	4175	4182	4190	4197	4205	33	26	6787	6794	6801	6808	6815	6823	33
27	4212	4220	4227	4235	4242	4250	32	27	6830	6837	6844	6851	6858	6865	32
28	4257	4264	4272	4279	4287	4294	31	28	6872	6879	6886	6893	6900	6907	31
29	4302	4309	4317	4324	4332	4339	30	29	6914	6921	6928	6935	6942	6950	30
30	9·9744 47	4354	4361	4369	4376	4384	29	30	9·9769 57	6964	6971	6978	6985	6992	29
31	4391	4399	4406	4414	4421	4428	28	31	6999	7006	7013	7020	7027	7034	28
32	4436	4443	4451	4458	4466	4473	27	32							
33	4481	4488	4495	4503	4510	4518	26	33							
34	4525	4533	4540	4547	4555	4562	25	34							
35	4570	4577	4585	4592	4599	4607	24	35	7107	7114	7121	7128	7135	7142	24
36	4614	4622	4629	4636	4644	4651	23	36	7209	7216	7223	7230	7237	7244	23
37	4659	4666	4674	4681	4688	4696	22	37	7251	7258	7265	7272	7279	7286	22
38	4703	4711	4718	4725	4733	4740	21	38	7293	7300	7307	7314	7321	7328	21
39	4748	4755	4762	4770	4777	4784	20	39	7335	7342	7349	7356	7363	7370	20
40	9·9747 92	4799	4807	4814	4821	4829	19	40	9·9773 77	7384	7391	7398	7405	7412	19
41	4836	4844	4851	4858	4866	4873	18	41	7419	7426	7433	7440	7447	7454	18
42	4880	4888	4895	4903	4910	4917	17	42	7461	7468	7475	7482	7489	7496	17
43	4925	4932	4939	4947	4954	4961	16	43	7503	7510	7517	7524	7530	7537	16
44	4969	4976	4984	4991	4998	5006	15	44	7544	7551	7558	7565	7572	7579	15
45	5013	5020	5028	5035	5042	5050	14	45	7586	7593	7600	7607	7614	7621	14
46	5057	5064	5072	5079	5086	5094	13	46	7628	7635	7642	7648	7655	7662	13
47	5101	5108	5116	5123	5130	5138	12	47	7669	7676	7683	7690	7697	7704	12
48	5145	5152	5160	5167	5174	5182	11	48	7711	7718	7725	7732	7739	7745	11
49	5189	5196	5204	5211	5218	5226	10	49	7752	7759	7766	7773	7780	7787	10
50	9·9752 33	5240	5248	5255	5262	5270	9	50	9·9777 94	7801	7808	7815	7821	7828	9
51	5277	5284	5292	5299	5306	5313	8	51	7835	7842	7849	7856	7863	7870	8
52	5321	5328	5335	5343	5350	5357	7	52	7877	7884	7890	7897	7904	7911	7
53	5365	5372	5379	5386	5394	5401	6	53	7918	7925	7932	7939	7946	7952	6
54	5408	5416	5423	5430	5437	5445	5	54	7959	7966	7973	7980	7987	7994	5
55	5452	5459	5467	5474	5481	5488	4	55	8001	8007	8014	8021	8028	8035	4
56	5496	5503	5510	5518	5525	5532	3	56	8042	8049	8056	8062	8069	8076	3
57	5539	5547	5554	5561	5568	5576	2	57	8083	8090	8097	8104	8110	8117	2
58	5583	5590	5598	5605	5612	5619	1	58	8124	8131	8138	8145	8152	8158	1
59	5627	5634	5641	5648	5656	5663	0	59	8165	8172	8179	8186	8193	8199	0
	60″	50″	40″	30″	20″	10″	Min.		60″	50″	40″	30″	20″	10″	Min.
	Co-sine of 19 Degrees.								Co-sine of 18 Degrees.						
P. Part	1″ 1	2″ 1	3″ 2	4″ 3	5″ 4	6″ 4	7″ 5	8″ 6	9″ 7	P. Part	1″ 1	2″ 1	3″ 2	4″ 3	5″ 4	6″ 4	7″ 5	8″ 6	9″ 6

Logarithm for 71°26’ = 9.976787

N.	0	1	2	3	4	5	6	7	8	9	D.
362	558709	8829	8948	9068	9188	9308	9428	9548	9667	9787	120
363	9907	..26	.146	.265	.385	.504	.624	.743	.863	.982	
364	561101	1221	1340	1459	1578	1698	1817	1936	2055	2174	119
365	2293	2412	2531	2650	2769	2887	3006	3125	3244	3362	
366	3481	3600	3718	3837	3955	4074	4192	4311	4429	4548	
367	4666	4784	4903	5021	5139	5257	5376	5494	5612	5730	118
368	5848	5966	6084	6202	6320	6437	6555	6673	6791	6909	
369	7026	7144	7262	7379	7497	7614	7732	7849	7967	8084	
370	8202	8319	8436	8554	8671	8788	8905	9023	9140	9257	117
371	9374	9491	9608	9725	9842	9959	..76	.193	.309	.426	
372	570543	0660	0776	0893	1010	1126	1243	1359	1476	1592	
373	1709	1825	1942	2058	2174	2291	2407	2523	2639	2755	116
374	2872	2988	3104	3220	3336	3452	3568	3684	3800	3915	
375	4031	4147	4263	4379	4494	4610	4726	4841	4957	5072	
376	5188	5303	5419	5534	5650	5765	5880	5996	6111	6226	115
377	6341	6457	6572	6687	6802	6917	7032	7147	7262	7377	
378	[illegible]	[illegible]	[illegible]	7836	7951	8065	8181	8295	8410	8525	
379	[illegible]	[illegible]	[illegible]	8983	9097	9212	9326	9441	9555	9669	114
380	[illegible]	[illegible]	[illegible]	.126	.241	.355	.469	.583	.697	.811	
381	580925	1039	1153	1267	1381	1495	1608	1722	1836	1950	
382	2063	2177	2291	2404	2518	2631	2745	2858	2972	3085	
383	3199	3312	3426	3539	3652	3765	3879	3992	4105	4218	113
384	4331	4444	4557	4670	4783	4896	5009	5122	5235	5348	
385	5461	5574	5686	5799	5912	6024	6137	6250	6362	6475	
386	6587	6700	6812	6925	7037	7149	7262	7374	7486	7599	112
387	7711	7823	7935	8047	8160	8272	8384	8496	8608	8720	
388	8832	8944	9056	9167	9279	9391	9503	9615	9726	9838	
389	9950	..61	.173	.284	.396	.507	.619	.730	.842	.953	
390	591065	1176	1287	1399	1510	1621	1732	1843	1955	2066	111
391	2177	2288	2399	2510	2621	2732	2843	2954	3064	3175	
392	3286	3397	3508	3618	3729	3840	3950	4061	4171	4282	
393	4393	4503	4614	4724	4834	4945	5055	5165	5276	5386	110
394	5496	5606	5717	5827	5937	6047	6157	6267	6377	6487	
395	6597	6707	6817	6927	7037	7146	7256	7366	7476	7586	
396	7695	7805	7914	8024	8134	8243	8353	8462	8572	8681	
397	8791	8900	9009	9119	9228	9337	9446	9556	9665	9774	109
398	9883	9992	.101	.210	.319	.428	.537	.646	.755	.864	
399	600973	1082	1191	1299	1408	1517	1625	1734	1843	1951	
400	2060	2169	2277	2386	2494	2603	2711	2819	2928	3036	108
401	3144	3253	3361	3469	3577	3686	3794	3902	4010	4118	
402	4226	4334	4442	4550	4658	4766	4874	4982	5089	5197	
403	5305	5413	5521	5628	5736	5844	5951	6059	6166	6274	
404	6381	6489	6596	6704	6811	6919	7026	7133	7241	7348	107
405	7455	7562	7669	7777	7884	7991	8098	8205	8312	8419	
406	8526	8633	8740	8847	8954	9061	9167	9274	9381	9488	
407	9594	9701	9808	9914	..21	.128	.234	.341	.447	.554	
408	610660	0767	0873	0979	1086	1192	1298	1405	1511	1617	106

N.	0	1	2	3	4	5	6	7	8	9	D.
Differences. Proportional Parts.	119	12	24	36	48	60	71	83	95	107	
	118	12	24	35	47	59	71	83	94	106	
	117	12	23	35	47	59	70	82	94	105	
	116	12	23	35	46	58	70	81	93	104	
	115	12	23	35	46	58	69	81	92	104	
	114	11	23	34	46	57	68	80	91	103	
	113	11	23	34	45	57	68	79	90	102	
	112	11	22	34	45	56	67	78	90	101	
	111	11	22	33	44	56	67	78	89	100	
	110	11	22	33	44	55	66	77	88	99	
	109	11	22	33	44	55	65	76	87	98	
	108	11	22	32	43	54	65	76	86	97	
	107	11	21	32	43	54	64	75	86	96	

Logarithm for 384 = .584331

N.	0	1	2	3	4	5	6	7	8	9	D.
944	974972	5018	5064	5110	5156	5202	5248	5294	5340	5386	46
945	5432	5478	5524	5570	5616	5662	5707	5753	5799	5845	
946	5891	5937	5983	6029	6075	6121	6167	6212	6258	6304	
947	6350	6396	6442	6488	6533	6579	6625	6671	6717	6763	
948	6808	6854	6900	6946	6992	7037	7083	7129	7175	7220	
949	7266	7312	7358	7403	7449	7495	7541	7586	7632	7678	
950	7724	7769	7815	7861	7906	7952	7998	8043	8089	8135	
951	8181	8226	8272	8317	8363	8409	8454	8500	8546	8591	
952	8637	8683	8728	8774	8819	8865	8911	8956	9002	9047	
953	9093	9138	9184	9230	9275	9321	9366	9412	9457	9503	
954	9548	9594	9639	9685	9730	9776	9821	9867	9912	9958	
955	980003	0049	0094	0140	0185	0231	0276	0322	0367	0412	45
956	0458	0503	0549	0594	0640	0685	0730	0776	0821	0867	
957	0912	0957	1003	1048	1093	1139	1184	1229	1275	1320	
958	1366	1411	1456	1501	1547	1592	1637	1683	1728	1773	
959	1819	1864	1909	1954	2000	2045	2090	2135	2181	2226	
960	2271	2316	2362	2407	2452	2497	2543	2588	2633	2678	
961	2723	2769	2814	2859	2904	2949	2994	3040	3085	3130	
962	3175	3220	3265	3310	3356	3401	3446	3491	3536	3581	
963	3626	3671	3716	3762	3807	3852	3897	3942	3987	4032	
964	4077	4122	4167	4212	4257	4302	4347	4392	4437	4482	
965	4527	4572	4617	4662	4707	4752	4797	4842	4887	4932	
966	4977	5022	5067	5112	5157	5202	5247	5292	5337	5382	
967	5426	5471	5516	5561	5606	5651	5696	5741	5786	5830	
968	5875	5920	5965	6010	6055	6100	6144	6189	6234	6279	
969	6324	6369	6413	6458	6503	6548	6593	6637	6682	6727	
970	6772	6817	6861	6906	6951	6996	7040	7085	7130	7175	
971	7219	7264	7309	7353	7398	7443	7488	7532	7577	7622	
972	7666	7711	7756	7800	7845	7890	7934	7979	8024	8068	
973	8113	8157	8202	8247	8291	8336	8381	8425	8470	8514	
974	8559	8604	8648	8693	8737	8782	8826	8871	8916	8960	
975	9005	9049	9094	9138	9183	9227	9272	9316	9361	9405	
976	9450	9494	9539	9583	9						
977	9895	9939	9983	..28	.						
978	990339	0383	0428	0472	0						
979	0783	0827	0871	0916	0						
980	1226	1270	1315	1359	1403	1448	1492	1536	1580	1625	
981	1669	1713	1758	1802	1846	1890	1935	1979	2023	2067	
982	2111	2156	2200	2244	2288	2333	2377	2421	2465	2509	
983	2554	2598	2642	2686	2730	2774	2819	2863	2907	2951	
984	2995	3039	3083	3127	3172	3216	3260	3304	3348	3392	
985	3436	3480	3524	3568	3613	3657	3701	3745	3789	3833	
986	3877	3921	3965	4009	4053	4097	4141	4185	4229	4273	
987	4317	4361	4405	4449	4493	4537	4581	4625	4669	4713	
988	4757	4801	4845	4889	4933	4977	5021	5065	5108	5152	
989	5196	5240	5284	5328	5372	5416	5460	5504	5547	5591	
990	5635	5679	5723	5767	5811	5854	5898	5942	5986	6030	
991	6074	6117	6161	6205	6249	6293	6337	6380	6424	6468	
992	6512	6555	6599	6643	6687	6731	6774	6818	6862	6906	
993	6949	6993	7037	7080	7124	7168	7212	7255	7299	7343	
994	7386	7430	7474	7517	7561	7605	7648	7692	7736	7779	
995	7823	7867	7910	7954	7998	8041	8085	8129	8172	8216	
996	8259	8303	8347	8390	8434	8477	8521	8564	8608	8652	
997	8695	8739	8782	8826	8869	8913	8956	9000	9043	9087	
998	9131	9174	9218	9261	9305	9348	9392	9435	9479	9522	
999	9565	9609	9652	9696	9739	9783	9826	9870	9913	9957	43

N.		0	1	2	3	4	5	6	7	8	9	D.
Differ.	46	P. Parts.	5	9	14	18	23	28	32	37	41	
	45		5	9	14	18	23	27	32	36	41	
	44		4	9	13	18	22	26	31	35	40	
	43		4	9	13	17	22	26	30	34	39	

Number for Logarithm 993214 = 9.84
(Falls between 993172 and 993216, therefore use 984).